

Type IIB Conifold Transitions in Cosmology

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Abstract

We study four-dimensional low energy effective actions for conifold transitions of Calabi-Yau spaces in the context of IIB supergravity. The actions are constructed by examining the mass of D3-branes wrapped on collapsing/expanding three-cycles. We then study the cosmology of the conifold transition, including consequences for moduli stabilization, taking into account the effect of the additional states which become light at the transition. We find, the degree to which the additional states are excited is essential for whether the transition is dynamically realized.

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1 Introduction

The problem of string theory vacuum degeneracy splits into two parts, namely the continuous degeneracy due to moduli fields and the discrete one due to the large number of different topologies. As is well-known by now there are a number of different topology-changing processes [1][2][3] in string theory which connect the moduli spaces associated with topologically different compactifications. A detailed understanding of these processes may perhaps be considered as a first step towards resolving the topological degeneracy. In this paper, we will study a certain type of topology-changing process, the conifold transition, as an explicitly time-dependent phenomena in the context of string cosmology.

A “milder” type of topology changing transition which arises in the context of type II Calabi-Yau compactifications is referred to as “flop”. In such a transition, a two-cycle within the Calabi-Yau space contracts to a point and re-expands as another two-cycle, thereby leading to a topologically distinct Calabi-Yau space with different intersection numbers. However, on either side of the transition the Hodge numbers are the same and so the spectrum of massless moduli is unchanged, while only their interactions are affected. The study of 5D cosmology associated with a flop of the internal space in M-theory was initiated in [4] and followed up in [5][6]. There it was found that the tension of any M2-branes wrapping the collapsing two-cycles caused the cycles to remain small, so the moduli forced the CY to remain near the flop point. Earlier work on black holes where the internal manifold undergoes a flop as a function of radius is given in [7].

In the conifold transition, which we study here, a three-cycle on one side of the transition collapses and then re-expands into a two-cycle, thus changing the Hodge numbers of the CY. This results in a different spectrum of massless moduli on either side of the transition, earning it the sobriquet “drastic” in [3]. At least in the case where the CY manifolds are quintics in \mathbb{CP}^4 the generic singularity is a conifold singularity and as such they form an important part of the set of singularities [8].

Although such conifold points naively give a singular low energy effective theory it is a remarkable property of string theory that they can be understood in a well defined manner[9]. When calculating the effective action for the low energy degrees of freedom one integrates out the heavy modes, replacing their effect by altered couplings between the light modes. It only makes sense to integrate out the modes which are heavier than those one is interested in. To do otherwise is inconsistent and leads to singularities. As explained in [9] it is precisely because some unseen light modes were being integrated out that singularities appeared in the effective action for a conifold transition. These extra modes correspond to D3-branes wrapping the collapsing cycles, as the cycles get smaller so these states become lighter and must not be integrated out.

In this paper we examine the cosmology following from the internal CY undergoing such a drastic topology change. The situation is rather different to the flop where the wrapping states are massive on either side of the flop, so that dynamically the CY will evolve to be near the flop. For the conifold one finds that the brane wrapping states pick up flat directions in their potential as the three-cycle turns into a two-cycle. This creates the possibility that the CY will dynamically evolve away from the conifold point, unlike the case of the flop transition. However, unless the evolution points exactly along the flat direction the fields explore more of the potential and are then typically forced back to the conifold point, as we shall see in the numerical examples presented later. We also note here that a D3-brane wrapping a three-cycle appears as a point particle in 4D, but if it wraps a two-cycle then it gives a string like solution. Indeed, these strings were proposed as a confinement mechanism in [10]. Although it was found in [11] that such strings cannot cause confinement it does show that the flop and the conifold transitions have rather different properties.

The paper proceeds as follows: We start by briefly describing the properties of the conifold

in section 2. Section 3 gives a description of how to reduce ten dimensional IIB supergravity to four dimensions, including the effect of D3-branes. This leads to an $\mathcal{N} = 2$ supergravity which is presented in Section 4. The cosmological implications of this model are presented in Section 4.2 and we conclude in Section 5.

2 A brief review of the conifold geometry

Before we discuss the low energy effective action resulting from a conifold transition between two CYs we give here a brief introduction to the conifold singularity, see [12] [13][1] [2][14] [15][16] for a more complete discussion. First we review some terminology. For a given CY, the position of a singularity which locally looks like the quadric in \mathbb{C}^4

$$P = \sum_{A=1}^{A=4} (W^A)^2 = 0 \quad (2.1)$$

is called a *node*. This is the equation for a conifold singularity[12]. It is clearly singular at $W^A = 0$ as $P = 0 = dP$ at that point. We also note that it describes a conical shape because if W^A lies on the conifold (2.1) then so does λW^A . To find the base of the cone we intersect it with an S^7 of radius r centred at the node,

$$\sum_{A=1}^{A=4} (W^A)^2 = 0, \quad \sum_{A=1}^{A=4} |W^A|^2 = r^2. \quad (2.2)$$

Writing $W^A = x^A + iy^A$ we find

$$x.x = y.y, \quad x.y = 0, \quad x.x + y.y = r^2. \quad (2.3)$$

$x.x = \frac{1}{2}r^2$ defines an S^3 of radius $r/\sqrt{2}$, and $y.y = \frac{1}{2}r^2$ combined with $x.y = 0$ gives an S^2 fibred over the S^3 . So, the base is S^3 fibred by S^2 . As such fibrations are trivial then the base of the conifold is the product $S^3 \times S^2$. The two distinct ways for making the conifold regular correspond to blowing up either the S^2 to give the (small) *resolution* or by blowing up the S^3 to give the *deformation*. The conifold transition then describes a CY going between these two regular manifolds. We denote the conifold by \mathcal{M}^\sharp , the resolved manifold by \mathcal{M}^b , and the deformed manifold by $\check{\mathcal{M}}$. A nice picture of the transition was presented in [12] and is given in Fig. 1. It shows the finite S^3 of the deformed conifold shrinking to zero and then being replaced by an expanding S^2 of the resolved conifold. This image will prove useful when we consider the dynamics of the transition in string theory.

Another part of the terminology is the *conifold point*, this being a location in the moduli space of CYs where the manifold acquires a node. In fact, we shall see that for the space to remain Kähler it must acquire a set of nodes [12]. Consider now a conifold containing P such nodes which have been deformed, thereby introducing P three-cycles. Not all of these need be homologically independent so we take there to be Q homology relations among them, giving $P - Q$ independent three-cycles. Now pick the standard homology basis for the independent three-cycles,

$$A^I . B_J = \delta_J^I, \quad I, J = 0, 1, 2, \dots h^{(2,1)}. \quad (2.4)$$

This introduces the *magnetic* cycles B_I , dual to the *electric* cycles A^I . In this work we shall consider the case where the collapsing cycles are composed solely of electric cycles in which case, because of the Q homology relations, each B_I intersects more than one collapsing cycle. Also, each vanishing

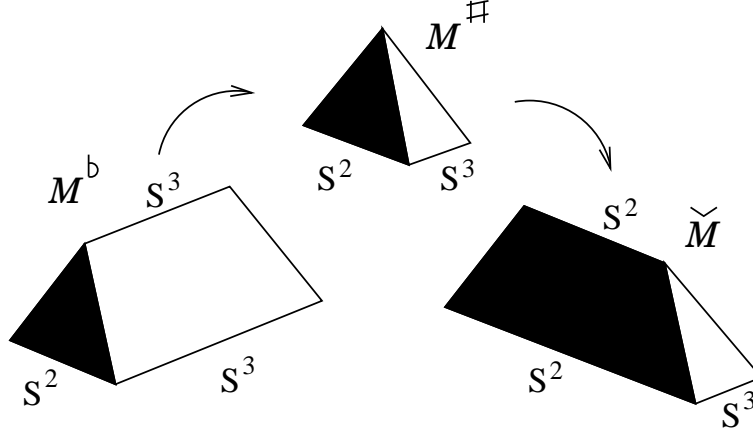


Figure 1: a representation of the conifold.

cycle must be involved in at least one homology relation if the manifold is to be Kähler, as will be seen later.

To see the effect of these homology relations we now follow a discussion of [10][13] and consider the relationship between A and B cycles. Fig. 1 shows a particular A -cycle three-sphere, A^1 , being blown down and then replaced by a two-sphere. The magnetic dual of this three-cycle is constructed as follows. The shaded region is the “cap” $\mathbb{R}_{\geq 0} \times S^2$, which can be completed into a three-cycle when extended away from the node. It is clear from the picture that this cycle intersects A^1 and can be chosen as its magnetic dual, B_1 . Also note that B_1 remains a three-cycle at the node, but when the node is resolved by a two-sphere then B_1 takes on the S^2 as a component of its boundary, so becomes a three-chain. Each A -cycle that B_1 intersects will provide an S^2 component to the boundary of B_1 and as such these two-spheres have a homology relation between them. Each of the magnetic cycles provides a homology relation between the two-spheres and so we find $P - Q$ relations between the two-spheres of the resolved manifold $\tilde{\mathcal{M}}$. That is, we get P two-cycles with $P - Q$ homology relations, i.e. Q independent two-cycles. The picture we are left with is illustrated in Fig. 2 where the magnetic cycle B^1 touches the three-cycles A^1, A^2, A^3 which shrink to zero and then turn into boundary two-spheres thereby converting the cycle B^1 into a chain. If a B -cycle had intersected only one A -cycle then in the resolved manifold this B -chain would have a single S^2 boundary so we would have for the Kähler form J ,

$$\begin{aligned} \int_{S^2} J &> 0, \\ \int_{\partial B} J &> 0, \\ \int_B dJ &> 0. \end{aligned} \tag{2.5}$$

Which violates the Kähler condition, $dJ = 0$. This is why each vanishing cycle must be involved in at least one homology relation. In particular this explains why a CY must have more than one node[12].

To re-cap then, we have that $P - Q$ independent three-spheres collapse and then expand as Q independent two-spheres. We can then see how this would appear from the point of view of the low energy effective theory. In this picture three-cycles correspond to complex structure moduli and each independent three-cycle gives a 4D vector-multiplet, while the two-cycles correspond to Kähler

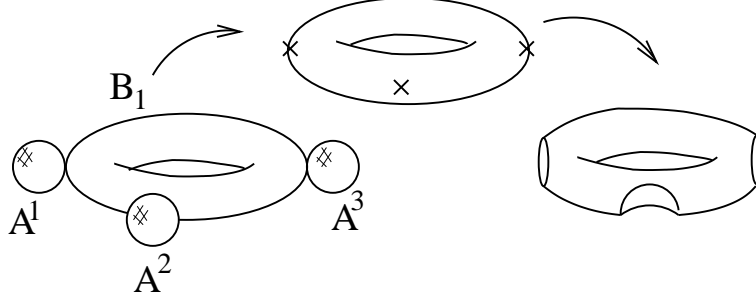


Figure 2: a representation of the cycles in a conifold transition.

moduli and each generates a 4D hyper-multiplet [17][18][19][20]. The conifold transition then sees $P - Q$ massless vector multiplets disappearing and Q massless hyper-multiplets appearing which is explained in the field theory picture as a Higgs phenomenon [10][8].

3 Reduction of IIB supergravity

In the framework of IIB supergravity we can dimensionally reduce on a CY-threefold to find 4D $\mathcal{N} = 2$ supergravity. The massless spectrum of this supergravity depends on the topology of the CY with the result that there are $h^{(2,1)}$ massless vector multiplets, $h^{(1,1)} + 1$ massless hyper-multiplets and one tensor multiplet [17][18][19][20]. As the inclusion of D3-branes is central to the resolution of the conifold singularity we also include the action of a probe D3-brane wrapping a three-cycle in the CY. We start with the IIB string-frame action given in [21], but with the RR fields rescaled by $\sqrt{2}$ for more standard conventions in 4D. The action is given by

$$S_{IIB}^{10} = \frac{1}{2K_{(10)}^2} \int \left[e^{-2\phi} \left(R \star 1 + 4d\phi \wedge \star d\phi - \frac{1}{2} H \wedge \star H \right) \right. \quad (3.6)$$

$$\left. -dl \wedge \star dl - \hat{F}_{(3)} \wedge \star \hat{F}_{(3)} - \frac{1}{2} \hat{F}_{(5)} \wedge \star \hat{F}_{(5)} - A_{(4)} \wedge H \wedge F_{(3)} \right],$$

$$S_{D3}^{10} = -\mu_3 \int_{D3} d^4 \xi e^{-\phi} \sqrt{-\det[P(g_{\mu\nu}^s)]} + \sqrt{2}\mu_3 \int_{D3} A_{(4)}, \quad (3.7)$$

where

$$F_{(3)} = dC_{(2)}, \quad \hat{F}_{(3)} = F_{(3)} - lH, \quad \hat{F}_{(5)} = dA_{(4)} - H \wedge C_{(2)}, \quad \mu_3 = \sqrt{\pi}/K_{(10)}. \quad (3.8)$$

The equation of motion and Bianchi identity for $\hat{F}_{(5)}$ are

$$d \star \hat{F}_{(5)} = H \wedge F_{(3)} = d\hat{F}_{(5)}, \quad (3.9)$$

so that we may consistently impose the required self-duality of the five-form field strength $\hat{F}_{(5)} = \star \hat{F}_{(5)}$.

We shall only be interested in the case where H , l and $C_{(2)}$ vanish. Then we can follow the standard procedure of expanding $A_{(4)}$ in terms of harmonic forms living on the CY,

$$A_{(4)} = V^I \wedge \alpha_I - U_I \wedge \beta^I, \quad (3.10)$$

$$\hat{F}_{(5)} = F^I \wedge \alpha_I - G_I \wedge \beta^I, \quad (3.11)$$

where we have introduced a basis of harmonic three-forms α_I, β^I and the one-forms V^I, U_I live on the spacetime and correspond to Abelian gauge fields with field strengths $F^I = dV^I, G_I = dU_I$. The harmonic forms are normalized to satisfy

$$\int_{A^I} \alpha_J = \int_{B_J} \beta^I = \sqrt{v} \delta_J^I, \quad \int_{CY} \alpha_I \wedge \beta^J = v \delta_I^J \quad (3.12)$$

with v being some reference CY volume. Self duality of $\hat{F}_{(5)}$ then relates G_I to F^I by

$$G_I = (ReM)_{IJ} F^J + (ImM)_{IJ} \star F^J, \quad (3.13)$$

where we define the matrices A, B, C, D, M [20]

$$\star \alpha_I = A_I^J \alpha_J + B_{IJ} \beta^J, \quad (3.14)$$

$$\star \beta^I = C^{IJ} \alpha_J + D^I_J \beta^J, \quad (3.15)$$

$$A = (ReM)(ImM)^{-1}, \quad (3.16)$$

$$B = -(ImM) - (ReM)(ImM)^{-1}(ReM),$$

$$C = (ImM)^{-1}.$$

The relation between G_I and F^I has important consequences for the interpretation of the charges of the particle states coming from wrapped branes. We shall see shortly that G_I is in fact the magnetic dual of F^I , so states charged under G_I are in fact magnetic charges of F^I .

With this decomposition of the field strengths we are in a position to dimensionally reduce the D3-brane action (3.7). To do so one first changes to the 10D Einstein frame and writes the metric in this frame as a direct product between a CY and a 4D spacetime. This gives an action in 4D which needs to be Weyl rescaled by the volume of the CY, κ , to get it in the 4D Einstein frame,

$$\kappa = \frac{1}{6} \int_{CY} J \wedge J \wedge J. \quad (3.17)$$

One then finds that [20]

$$S_{IIB}^4 = \frac{1}{K_{(4)}^2} \int \left[\frac{1}{2} R \star 1 - g_{i\bar{j}} dz^i \wedge \star dz^{\bar{j}} - h_{uv} dq^u \wedge \star dq^v \right. \quad (3.18)$$

$$\left. + \frac{1}{2} F^I \wedge (Im(M)_{IJ} \star F^J + Re(M)_{IJ} F^J) \right],$$

$$K_{(4)}^2 = K_{(10)}^2 / v, \quad (3.19)$$

$$i, j = 1, 2 \dots h^{2,1}, \quad I, J = 0, 1 \dots h^{2,1}, \quad (3.20)$$

where the complex structure moduli, z^i , have a moduli metric $g_{i\bar{j}}$ derived from the Kähler potential $K_{(\Omega)}$,

$$e^{-K_{(\Omega)}} = (i/v) \int_{CY} \Omega \wedge \bar{\Omega} = \|\Omega\|^2 \kappa / v, \quad (3.21)$$

and the fields q^u are the hyper-multiplets which have a quaternion-Kähler moduli metric h_{uv} . Written in this form, (3.18) shows that the field strength G_I (3.13) is the magnetic dual of F^I as was claimed earlier.

Along with the reduction of the supergravity we also need to dimensionally reduce the D3-brane action (3.7), but to do that we need to know more about the cycle it is wrapping. Quite generally we may decompose this cycle \mathcal{C} as

$$\mathcal{C} = n_I A^I + m^I B_I, \quad n_I, m_I \in \mathbb{Z}. \quad (3.22)$$

The extra piece of information which we need is that the D3-brane lives on a minimal, supersymmetric SLAG three-cycle [22][23][24]. Such cycles are calibrated by $Re(e^{i\theta}\Omega)$, for some constant θ , and saturate the following bound

$$Vol(\mathcal{C}) \geq \frac{\sqrt{\kappa} \left| \int \Omega \right|}{\left| \int \Omega \wedge \bar{\Omega} \right|} = \sqrt{(\kappa/v)} e^{(K_{(\Omega)}/2)} \left| \int \Omega \right|. \quad (3.23)$$

By writing the holomorphic three-form as

$$\Omega = X^I \wedge \alpha_I - \mathcal{F}_I \wedge \beta^I, \quad (3.24)$$

we find

$$e^{-K_\Omega} = \frac{i}{v} \int \Omega \wedge \bar{\Omega} = i (\mathcal{F}_I \bar{X}^I - X^I \bar{\mathcal{F}}_I), \quad (3.25)$$

and by wrapping the brane on a SLAG cycle we can perform the spatial integration in (3.7) to find that the D3-brane action becomes, after the Weyl re-scalings,

$$S_{D3}^4 = -\frac{\sqrt{\pi}}{K_{(4)}} e^{(K_\Omega/2)} |n_I X^I - m^I F_I| \int d\tau + \frac{\sqrt{2\pi}}{K_{(4)}} n_I \int V^I - \frac{\sqrt{2\pi}}{K_{(4)}} m^I \int U_I. \quad (3.26)$$

This corresponds to a particle of mass $\frac{\sqrt{\pi}}{K_{(4)}} e^{(K_\Omega/2)} |n_I X^I - m^I F_I|$, charged under the gauge fields U_I and V_I . Here, τ is the proper time of the particle. This relation between mass and charge is what is to be expected for an $\mathcal{N} = 2$ supergravity [25]. As explained earlier, electric charges of U_I are in fact magnetic charges of V^I . This explains why the A^I and B_I cycles were termed electric and magnetic respectively. We shall only be interested in cases where the particles have electric charge so we set m^I to zero and only wrap the D3-brane over the A^I .

4 $\mathcal{N} = 2$ supergravity

The way we proceed is to use the structure of $\mathcal{N} = 2$ supergravity along with the information we have about the extra brane wrapping states to write down the 4D effective action of the conifold transition. We still have $h^{(2,1)}$ vector multiplets but now there are $h^{(1,1)} + 1 + P$ hyper multiplets, corresponding to the usual set plus P from wrapped three-cycles. Due to the Q homology relations these extra hyper multiplets are charged under $P - Q$ of the vector fields. What we require then is a gauged supergravity to account for hyper-multiplets charged under Abelian gauge groups. Taking results from [26][27] and using our conventions we have

$$\begin{aligned} S_{\mathcal{N}=2}^4 = & \frac{1}{K_{(4)}^2} \int \left[\frac{1}{2} R \star 1 - g_{i\bar{j}} dz^i \wedge \star dz^{\bar{j}} - h_{uv} Dq^u \wedge \star Dq^v \right. \\ & + \frac{1}{2} Im(\mathcal{N})_{IJ} F^I \wedge \star F^J - \frac{1}{2} Re(\mathcal{N})_{IJ} F^I \wedge F^J \\ & \left. - g^2 \left[4h_{uv} k_I^u k_J^v \bar{L}^I L^J + (g^{i\bar{j}} f_i^I f_{\bar{j}}^J - 3\bar{L}^I L^J) \mathcal{P}_I^x \mathcal{P}_J^x \right] \right], \end{aligned} \quad (4.27)$$

$$Dq^u = dq^u + gA^I k_I^u(q), \quad (4.28)$$

$$L^I = e^{(K(\Omega)/2)} X^I, \quad (4.29)$$

$$\mathcal{F}_I = \mathcal{N}_{IJ} X^J, \quad (4.30)$$

$$f_i^I = \left(\partial_i + \frac{1}{2} \partial_i K(\Omega) \right) e^{\frac{1}{2} K(\Omega)} X^I, \quad (4.31)$$

$$\bar{f}_{\bar{i}}^I = \left(\partial_{\bar{i}} + \frac{1}{2} \partial_{\bar{i}} K(\Omega) \right) e^{\frac{1}{2} K(\Omega)} \bar{X}^I \quad (4.32)$$

$$\exp(-K(\Omega)) = i(\bar{X}^I \mathcal{F}_I - X^I \bar{\mathcal{F}}_I), \quad (4.33)$$

$$k_I \lrcorner K^x = \nabla \mathcal{P}_I^x. \quad (4.34)$$

The X^I are holomorphic functions of the z^i and if we use special co-ordinates then we have that the complex structure part of the action is determined by a pre-potential, \mathcal{F} , which is a degree two holomorphic function of X^I . The Kähler potential (4.34) is then determined in terms of the first derivatives

$$\mathcal{F}_I = \frac{\partial \mathcal{F}}{\partial X^I}, \quad (4.35)$$

and can be obtained as a function of z^i by setting $X^I = (1, z^i)$. The matrix \mathcal{N}_{IJ} is known as the period matrix and is given by [25][26][27]

$$\mathcal{N}_{IJ} = \bar{\mathcal{F}}_{IJ} + \frac{2i(\text{Im}\mathcal{F}_{IK})X^K(\text{Im}\mathcal{F}_{JL})X^L}{(\text{Im}\mathcal{F}_{MN})X^M X^N} \quad (4.36)$$

where

$$\mathcal{F}_{IJ} = \partial_I \partial_J \mathcal{F} \quad (4.37)$$

Comparison of action (4.27) with (3.18) shows that this period matrix equates to the matrix M_{IJ} introduced in (3.16). This action contains two important metrics, $g_{i\bar{j}}$ and h_{uv} . The gauging of the scalars comes by gauging with respect to isometries of these metrics. Here we are only interested in giving the hyper-multiplets charge with respect to an Abelian group and so we have restricted the action of [26][27] to give the above form, with k_I^u being the Killing vectors on h_{uv} . Further, \mathcal{P}_I^x are the associated pre-potentials and the triplet of two-forms K^x represent the hyper-Kähler forms. The operator ∇ appearing in (4.34) is the SU(2) covariant derivative on h_{uv} .

4.1 An example

To make any progress we need to pick a particular CY to reduce on. This is equivalent to choosing a pre-potential for the complex structure and a quaternionic metric for the hyper-multiplets. The exact quaternionic sigma model manifold for the brane wrapping hypermultiplets is unknown as yet and will in general mix the wrapping-hypermultiplet components with the Kähler moduli. This is necessary because the overall hypermultiplet manifold must be quaternionic, and as such cannot be a direct product of manifolds [5]. However, we know from the brane reduction that the mass term for the hypermultiplets only involves complex structure terms and not the Kähler moduli, this means that to leading order we can neglect the coupling between the Kähler moduli and the hypermultiplets representing the wrapping states. As such, as long as the hypermultiplets remain small we can consider the approximation that the quaternionic metric is flat and switch off all hypermultiplets other than the wrapping states. Higher order terms will couple the brane wrapping states to the usual hyper-multiplets forming the full quaternionic manifold. However, we do not

expect the higher order terms to alter the qualitative behaviour of the dynamics. For example, the initial study of [4] on flop transitions made this same approximation which was amended in [6][5] but lead to the same structure. More recently, dynamics of conifolds transitions in the context of M-theory were examined with both a first order approximation and using complete Wolf spaces [34]; the same conclusion was reached. With this understood we will set $h_{uv} = \delta_{uv}$ and so the ∇ in (4.34) gets replaced by the usual exterior derivative. The simulations in the following section will reveal that the q^{au} remain small in our examples, maintaining the validity of our approximation.

Now we must construct the Killing vectors which are used to gauge the hyper-multiplets. We shall denote the hyper-multiplets by indices $a, b = 1, 2, \dots, P$ where we have switched off the other $h^{(1,1)} + 1$ hyper-multiplets which come from the usual supergravity reduction. This is consistent with the first order approximation we are using for the hyper-multiplet metric. The components of a given hyper-multiplet are labelled with the indices $u, v = 1, 2, 3, 4$ so we write q^{au} for hyper-multiplet components, their charge is given in units of g as gQ_I^a . The gauge fields take on the labels $I, J = 0, 1, \dots, h^{(2,1)}$ but for Q homology relations among the P wrapped cycles only $P - Q$ of these gauge fields are used in the gauging of q^{au} . As our metric is taken to be flat we have a choice of rotation or translation Killing vectors. For the potential to vanish when the q^{au} do then the rotation Killing vectors are the relevant choice,

$$k_I^{au} = \sum Q_I^a t^u{}_v q^{av}, \quad (4.38)$$

where t is an anti-symmetric matrix. Moreover, we can introduce the 't Hooft symbols [28].

$$\eta_{bc}^a = \bar{\eta}_{bc}^a = \epsilon_{bc}^a, \quad (4.39)$$

$$\eta_{b0}^a = \bar{\eta}_{b0}^a = \delta_b^a, \quad (4.40)$$

and write a set of hyper-Kähler forms as

$$K_{au,bv}^x = -\bar{\eta}_{uv}^x \delta_{ab}. \quad (4.41)$$

By decomposing the matrix t as

$$t_{uv} = n_x \eta_{uv}^x, \quad (4.42)$$

where \underline{n} is a unit vector, we find the following pre-potential for the Killing vectors,

$$\mathcal{P}_I^x = \frac{1}{2} \sum Q_I^a q^{av} (\bar{\eta}_{uv}^x n_y \eta_w^{y,u}) q^{aw}. \quad (4.43)$$

Here we have dropped the possible constants of integration (Fayet-Illiopoulos terms) as they would give a potential for the complex structure even in the absence of brane wrapping i.e. $q^u = 0$. Then the Killing vector terms in the potential become

$$V_{IJ}^{(D)} = P_I^x P_J^x = \sum Q_I^a Q_J^b \left(\frac{1}{2} q^{av} q^{aw} q^{bv} q^{bw} - \frac{1}{4} q^{av} q^{av} q^{bw} q^{bw} + \frac{1}{2} q^{av} q^{aw} q^{br} q^{bt} t_{wt} t_{vr} \right), \quad (4.44)$$

$$V_{IJ}^{(m)} = h_{au,av} k_I^{au} k_J^{av} = \sum Q_I^a Q_J^a q^{aw} q_{aw}. \quad (4.45)$$

The quadratic part of the hyper-multiplet Lagrangian may then be extracted to find the their mass in terms of g , then using the earlier result that branes wrapped on electric cycles have mass $\frac{\sqrt{\pi}}{K_{(4)}} e^{(K_{\Omega/2})} |n_I X^I|$ we conclude that

$$g^2 = \pi / (4K_{(4)}^2). \quad (4.46)$$

To fix the rest of the model we need to choose a pre-potential \mathcal{F} , to do this we take the generic large complex structure form

$$\mathcal{F} = d_{IJK} \frac{X^I X^J X^K}{X^0}, \quad (4.47)$$

for constant d_{IJK} , and restrict to the more manageable case

$$d_{IJ0} = -\frac{i}{2} N_{IJ}, \quad N_{0i} = 0, \quad d_{IJi} = 0 \quad (4.48)$$

where we have introduced the constant coupling matrix N_{IJ} . Of course, the above form of the pre-potential is generically invalidated as some of the three-cycles become small near the conifold transition. However, corrections to the pre-potential will come in the form of logarithmic terms originating from the monodromy about the conifold point and other terms which are analytic at the conifold point[9]. The logarithmic terms are accounted for through the inclusion of the states that become light, and corrections which are not of the form (4.48) will be cubic and higher order; these will be negligible very close to the conifold point where the periods are vanishing. Hence we expect that they will not affect our qualitative conclusions and as a first approximation we do not include them. With this understood, we find that our 4D effective action becomes

$$\begin{aligned} S_{IIB+D3}^{(4)} = \frac{1}{K_4^2} \int \sqrt{-g} d^4x \left\{ \frac{1}{2} R_4 - \left(\frac{-N_{ij}}{\langle X|X \rangle} + \frac{z^m \bar{z}^n N_{mj} N_{ni}}{\langle X|X \rangle^2} \right) \partial_\mu z^i \partial^\mu \bar{z}^j \right. \\ - \nabla_\mu q^{au} \nabla^\mu q_{au} - 2g^2 \left(\frac{z^i \bar{z}^j}{\langle X|X \rangle} \right) V_{ij}^{(m)} \\ - g^2 \left(-\frac{1}{2} (N^{-1})^{ij} - \frac{\bar{z}^i z^j}{\langle X|X \rangle} \right) V_{ij}^{(D)} \\ \left. - \frac{1}{4} \text{Im}(\mathcal{N})_{IJ} F_{\mu\nu}^I F^{J\mu\nu} \right\}. \end{aligned} \quad (4.49)$$

where

$$\langle X|X \rangle = N_{IJ} X^I \bar{X}^J = N_{00} + N_{ij} z^i \bar{z}^j \quad (4.50)$$

One finds that in order to have positive kinetic terms for the scalar fields while still satisfying (4.33) the coupling matrix N_{IJ} must have signature $(+,-,-,\dots)$ [25][26][27].

4.2 Cosmology of conifolds.

As we are interested in the cosmological evolution following from a conifold transition we shall take all the scalar fields to depend only on time. The vector fields vanish by isotropy considerations. If we take the spacetime metric to Friedmann-Robertson-Walker with flat spatial sections,

$$ds^2 = -dt^2 + a(t)^2 d\underline{x}^2, \quad (4.51)$$

then we find the following equations of motion from (4.49).

$$\ddot{q}_{av} + 3 \left(\frac{\dot{a}}{a} \right) \dot{q}_{av} + \frac{1}{2} \partial_a V = 0, \quad (4.52)$$

$$\ddot{z}^i + 3 \left(\frac{\dot{a}}{a} \right) \dot{z}^i + \Gamma^i_{jk} \dot{z}^j \dot{z}^k + \partial_{\bar{j}} V g^{\bar{j}i} = 0, \quad (4.53)$$

$$Q_I^a t^u{}_v q^{av} \partial_\mu q_{au} = 0, \quad (4.54)$$

$$2 \left(\frac{\ddot{a}}{a} \right) + \left(\frac{\dot{a}}{a} \right)^2 = -g_{i\bar{j}} \dot{z}^i \dot{\bar{z}}^j - \dot{q}^{av} \dot{q}_{av} + V, \quad (4.55)$$

$$3 \left(\frac{\dot{a}}{a} \right)^2 = g_{i\bar{j}} \dot{z}^i \dot{\bar{z}}^j + \dot{q}^{av} \dot{q}_{av} + V \quad (4.56)$$

where

$$V = 2g^2 \left(\frac{z^i \bar{z}^j}{\langle X|X \rangle} \right) V_{ij}^{(m)} + g^2 \left(-\frac{1}{2}(N^{-1})^{ij} - \frac{\bar{z}^i z^j}{\langle X|X \rangle} \right) V_{ij}^{(D)} \quad (4.57)$$

The connection on the complex structure moduli space is given by

$$\Gamma^i_{jk} = g^{\bar{l}i} \partial_j g_{k\bar{l}}. \quad (4.58)$$

When performing the numerical simulation, the gauge field equation (4.54) and the energy conservation equation (4.56) act as a check that the simulations are accurate. Also note that (4.54) is expressing the statement that there are no electric currents. In particular, for the cosmological simulations we have zero charge density, which corresponds to no net brane wrapping in the string theory picture. We shall perform our simulations in units where $\sqrt{\pi}/(2K_{(4)}) = 1$.

An interesting effect with the flop transition, noted in [4], is that even the moduli which are not involved in the flop were found to be stabilized once the additional hyper-multiplets acquire a non-zero value. Here we note a similar effect is possible depending on the choice for the coupling matrix N_{IJ} . To see this let us consider the equation of motion for the complex structure (4.53) using (4.48)

$$\ddot{z}^i + 3 \left(\frac{\dot{a}}{a} \right) \dot{z}^i + \Gamma^i_{jk} \dot{z}^j \dot{z}^k - (N^{-1})^{ij} z^k \left(V_{jk}^{(m)} - V_{jk}^{(D)} \right) = 0. \quad (4.59)$$

As hyper-multiplets are only charged under the gauge fields associated with the wrapped three-cycles, then one can easily show that those z^i where the index i corresponds to an unwrapped cycle can be a flat directions in the potential; $\partial_{\bar{j}} g^{\bar{j}i} V = 0$. To see this, note from (4.44)(4.45) that $V_{jk}^{(m)}$, $V_{jk}^{(D)}$ vanish for those indices j, k not associated to the charges. A particularly simple choice for N_{IJ} is minimal coupling, with $N_{IJ} = \eta_{IJ}$ [29] giving

$$N_{00} = 1 \quad N_{i0} = 0 \quad N_{ij} = -\delta_{ij}. \quad (4.60)$$

For this case (4.59) gives a vanishing $\partial_{\bar{j}} g^{\bar{j}i} V$ for those i which have $Q_i^a = 0$. We could say that in the case of minimal coupling only the wrapped z^i have a potential. If we think of the period matrix as the means by which the different vector multiplets couple to each other then allowing off diagonal terms in N_{ij} lets the unwrapped z^i communicate with the wrapped ones, so in this case if the moduli for the wrapped cycles get stabilized there is also the possibility that the moduli for unwrapped cycles will get stabilized. Indeed, from (4.59) we see that for non-minimal coupling (allowing off-diagonal terms in N_{ij}) even the z^i which are unwrapped ($Q_i^a = 0$) have a forcing term which is linear in z , corresponding to a quadratic potential. In our numerical experiments we present an example for a particular non-minimal period matrix where all of the z^i get stabilized. It is important to note that at the conifold point, or when no cycles are wrapped, the potential for all the moduli vanishes. Similarly on the Higgs branch only the wrapped moduli have a potential. And so we see that eventually the unwrapped moduli will return to be flat directions, but with an increased probability of being found near the location where the potential had a minimum during the transition. If both z^i and q^{au} are non-zero then they generate an effective mass for each other and so each are driven to oscillate about zero. Because of the cosmological expansion these

Plot	z	q^{11}	q^{12}	q^{13}	q^{14}	q^{21}	q^{22}	q^{23}	q^{24}
1 st	0.01	0.1	0.15	0.2	0.25	0.15	0.2	0.25	0.3
2 nd	0.95	1 ⁻⁴	1.5x10 ⁻⁴	2x10 ⁻⁴	2.5x10 ⁻⁴	2x10 ⁻⁴	3x10 ⁻⁴	4x10 ⁻⁴	5x10 ⁻⁴
3 rd	0.65	0.01	0.015	0.02	0.025	0.02	0.03	0.04	0.05

Table 1: Table showing the initial values for the fields, corresponding to the plots in Figure 3

oscillations are damped and we have several possibilities: The z^i could reach zero first leaving the q^{au} to wander in their flat directions and forcing the field theory into the Higgs phase; the q^{au} could vanish first in which case the z^i will not be driven to zero and the field theory remains in the Coulomb phase; or both the complex structure and the hyper-multiplets go to zero and the CY remains near the conifold point. Which of these possibilities actually occurs clearly depends on the initial conditions of the fields and we present numerical examples of all three scenarios. Note that it is consistent for the z^i to remain non-zero while the q^{au} decay because we have zero net brane wrapping, and so, we believe, the decay of q^{au} while the three-cycle is non-vanishing can be interpreted as the annihilation of brane anti-brane pairs. The initial values for the fields are shown in Table 1 and we take vanishing time derivatives for all the scalars as initial conditions. The evolution of the fields z , q^{11} , and q^{21} is plotted in Fig. 3. The scale factor has not been plotted as this just increased monotonically from its initial value of unity. Neither did we plot all of the components of the hyper-multiplets, these behaved in a similar manner to the first component so, to keep the plots simple, they are not shown.

For our first simulation we shall consider the case where the CY has two collapsing three-cycles such that their sum is homologically trivial. In the field theory this corresponds to two hyper-multiplets, q^{1u} and q^{2u} , charged under a single gauge field A^1 with opposite charges, which we take as $Q_1^1 = 1$, $Q_2^1 = -1$. We can now see explicitly how the flat directions appear once the three-cycles have collapsed, $z^1 \equiv z = 0$, by examining the potential in (4.27). The only term which survives is the term quadratic in the pre-potentials \mathcal{P}_I^x , and in the case of minimal coupling this term is $\delta^{ij} \mathcal{P}_i^x \mathcal{P}_j^x$ ¹ which is positive definite and vanishes when \mathcal{P}_i^x does. By examining (4.43) we see then that for $q^{1u} = q^{2u}$ then $\mathcal{P}_i^x = 0$, so our flat direction drives the components of the two hyper-multiplets towards each other. It is also trivial to see from (4.49) that if the q^{au} vanish then the complex structure scalars z^i have no potential and so become flat directions. The top two plots of Fig. 3 give the results of a run showing how the first components of two different hypermultiplets get driven toward each other as z remains small. This confirms the expectation that the hyper-multiplets follow the flat directions present at $z = 0$, so placing the gauge theory in the Higgs phase and allowing the conifold transition to complete.

In the second plot we see the case where the hyper-multiplets have reached small values before the complex structure modulus has reached the conifold point. In this case the potential generated by the q^{au} is too small to drive the z to zero and the field theory never reaches the conifold, leaving it in the Coulomb phase. This crucially depends on the Hubble damping being efficient enough to stop the q^{au} from oscillating. The third plot shows results corresponding to a case where both the complex structure and the hyper-multiplets oscillate around zero. From the string theory picture, this scenario describes the CY being stuck at the conifold point, with the oscillations of z keeping q small and vice versa. This means that both the three-cycles of the original CY and the two-cycles of the topologically related CY remain small.

As discussed earlier it is possible to couple the gauge field super-partners of the wrapped moduli to the gauge fields associated with unwrapped moduli through off diagonal elements in the coupling

¹ $\mathcal{P}_0^x = 0$

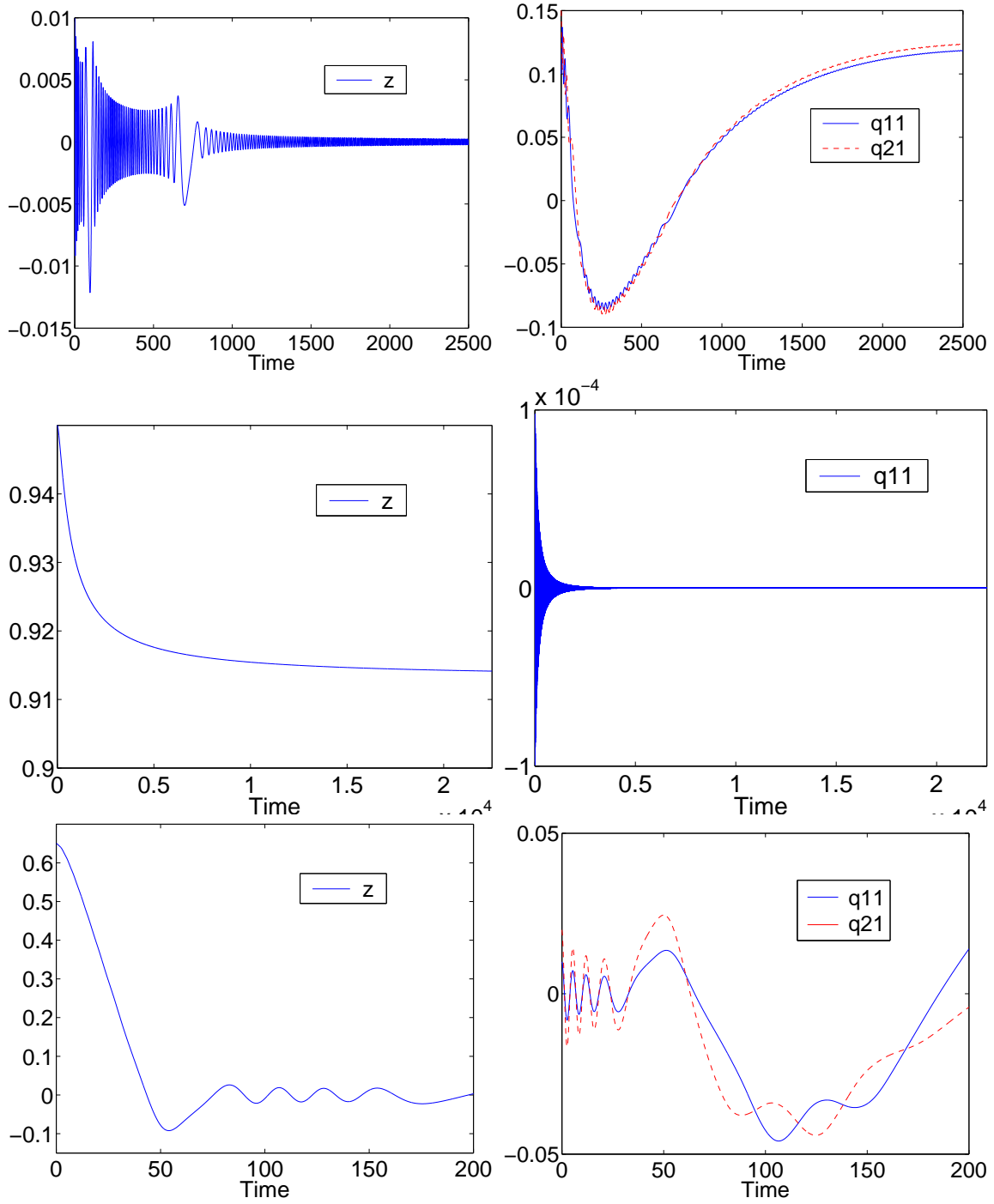


Figure 3: Figure showing the evolution of the fields $Re(z)$, q^{11} and q^{21} against time.

Field	z^1	z^2	q^{11}	q^{12}	q^{13}	q^{14}	q^{21}	q^{22}	q^{23}	q^{24}
Value	0.5	0.5	0.01	0.015	0.02	0.025	0.015	0.02	0.025	0.03

Table 2: Table showing the initial values of the fields plotted in Figure 4.

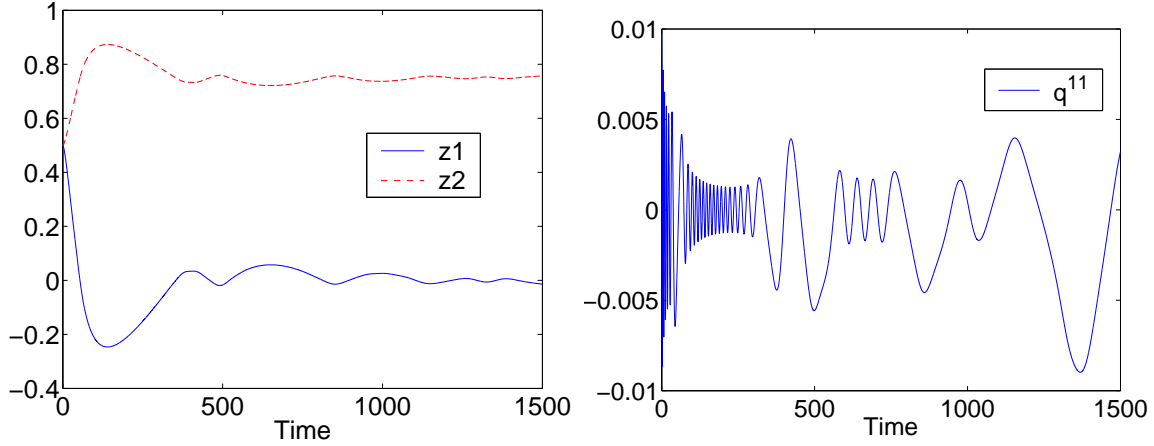


Figure 4: Plots showing evolution of q^{11} , the wrapped modulus $Re(z^1)$ and unwrapped modulus $Re(z^2)$ with initial values given in Table 2.

matrix, and we argued that this created dynamics for those cycles which were not even being wrapped. To give an example of this consider an unwrapped modulus z^2 along with the original $z = z^1$, but now we move away from minimal coupling, taking the coupling matrix to be of the form

$$N_{IJ} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & -1 \end{pmatrix}. \quad (4.61)$$

Fig. 4 shows the evolution of both the complex structure moduli z^1 , z^2 , along with the representative component q^{11} for the initial conditions given in Table 2. We see that, as expected, the evolution of the wrapped cycle corresponding to z^1 has created a potential for the unwrapped cycle given by z^2 . In this particular case the unwrapped cycle has also settled down to a particular size with its final value clearly depending on the choice of coupling matrix, and hence on the CY, as well as on the initial conditions. For this particular example the final state sees the hypermultiplets remaining small along with the value of z^1 , the wrapped cycle. That is to say, the fields remain at the conifold point.

5 Conclusion

In this paper we have examined the effect of a particular topological transition of the internal manifold in a Calabi-Yau compactification of IIB string theory. The transition in question is the conifold transition where one Calabi-Yau gets transformed to another via the shrinking and expanding of two-cycles and three-cycles. The question we try to answer here is how do these cycles evolve in a cosmological context? And is it possible to fix the moduli associated with these cycles?

By constructing the effective action for IIB supergravity with D3-branes wrapping the three-cycles we have seen that the brane wrapping states can be described by charged hyper-multiplets,

q^{au} , in four dimensions [9] with the charges corresponding to the cycles which are being wrapped. As was to be expected, exciting the wrapping states drove the three-cycles toward zero volume by creating a potential for associated complex structure moduli, z^i . However, once the three-cycles are at zero volume then the q^{au} have flat directions so could pick up a vev, placing the gauge theory in the Higgs phase and taking the Calabi-Yau successfully through the conifold transition. It was also possible that the wrapped D3-branes/anti-branes could annihilate before the three-cycles are able to reach the conifold point, in which case there is no longer a force driving the cycles to collapse and the theory remains in the Coulomb phase. A third possibility is that both the three-cycles and two-cycles are driven to zero dynamically, so that the Calabi-Yau ends up near the conifold point. By numerically evolving the equations of motion for the effective action we have seen that all three cases are possible, depending on initial conditions of the fields. In the absence of information about the processes causing the brane wrapping to occur it is impossible to state which outcome is more likely. However, it seems that in the case where the fields are of the same order of magnitude the generic evolution consists of the fields oscillating around zero. In this sense we would expect the internal space to typically be near the transition in a cosmological context. Taking the point of view given in [30] that quantum fields are always excited one would again conclude that it is more likely that the fields should remain near the conifold point. We note also that this picture makes our approximation of small q^{au} consistent and so we expect the quaternion-Kähler metric h_{uv} to be well described by the flat metric we used. This raises another issue, namely what are the typical values that these scalar fields should take? This is clearly an important point as the values of these fields can dramatically change the evolution of the system.

We have also seen how the structure of the Calabi-Yau, through its coupling matrix, can create a potential even for those cycles which are not being wrapped. Depending on the form of this matrix then, it is possible for these other complex structure moduli to be involved in the evolution.

As we have performed the simulations in a cosmological context, the oscillations of the fields have been damped by Hubble friction. This is in fact crucial if, for example, one wants to complete the conifold transition and end up in the Higgs phase. Without Hubble damping both the complex structure and the hyper-multiplets oscillate about zero in an apparently chaotic manner, thereby never reaching the state $z^i = 0$, $q^{au} \neq 0$, which would be a completed conifold transition. Hence, dynamically we would not expect a conifold transition to occur in Minkowski space.

In our effective theory we have made a rather arbitrary choice for pre-potential \mathcal{F} and not made any attempt to connect it to an existing Calabi-Yau. It would be a worthwhile exercise to make a more precise link with string theory and use results for a particular Calabi-Yau to tell us which \mathcal{F} to use. It would also be useful to have a clearer understanding of the role the coupling matrix plays in generating dynamics for those cycles which are not wrapped. Something which we have not touched upon is that of causality and the Kibble mechanism [31]. In the real Universe we only expect homogeneity on sufficiently large scales while on smaller scales we would expect the fields to take on different value as required by causality [31]. What this means for the conifold transition is that after its completion the broken U(1) gauge symmetry will generate cosmic strings. However, as emphasized in [11] these strings are rather different to the usual Nielsen-Olesen vortices [32] in that they are unstable to expansion of their core. It would therefore be interesting to study the dynamics of these strings to see how the instability-timescale alters the formation of string networks.

Note added While this paper was being prepared some related work was presented on the archive which also analyzes the cosmology of conifold transitions, albeit in a different context [33][34]. These

papers study the five-dimensional cosmology following from M-theory on a Calabi-Yau space as it goes through a conifold transition, including the effect of M2-branes wrapped on two-cycles.

Acknowledgements A. L. and P. M. S. are supported by PPARC Advanced Fellowships. E. P. is supported by a PPARC studentship.

Appendices

A Conventions

In this paper we take the metric to have signature $(-+++ \dots)$. We define the Levi-Civita tensor ϵ and the volume form η as

$$\epsilon_{012\dots} = +1 \tag{A.1}$$

$$dx^\mu \wedge dx^\nu \wedge \dots = -\epsilon^{\mu\nu\dots} \eta \tag{A.2}$$

$$\eta = dx^0 \wedge dx^1 \wedge \dots \tag{A.3}$$

The Hodge dual is defined as

$$\star F_p = \frac{1}{p!(d-p)!} F_{\mu_1\mu_2\dots\mu_p} \epsilon^{\mu_1\mu_2\dots\mu_p}{}_{\mu_{p+1}\mu_{p+2}\dots\mu_d} dx^{\mu_{p+1}} \wedge dx^{\mu_{p+2}} \wedge \dots dx^{\mu_d}. \tag{A.4}$$

Giving for p-forms α and β

$$\alpha \wedge \star \beta = (\alpha \lrcorner \beta) \eta = \frac{1}{p!} \alpha^{\mu\nu\dots} \beta_{\mu\nu\dots} \eta \tag{A.5}$$

References

- [1] P. Candelas, P. S. Green and T. Hubsch, Phys. Rev. Lett. **62** (1989) 1956.
- [2] P. Candelas, P. S. Green and T. Hubsch, Nucl. Phys. B **330** (1990) 49.
- [3] B. R. Greene, arXiv:hep-th/9702155.
- [4] M. Brandle and A. Lukas, Phys. Rev. D **68** (2003) 024030 [arXiv:hep-th/0212263].
- [5] L. Jarv, T. Mohaupt and F. Saueressig, JHEP **0312** (2003) 047 [arXiv:hep-th/0310173].
- [6] L. Jarv, T. Mohaupt and F. Saueressig, JCAP **0402** (2004) 012 [arXiv:hep-th/0310174].
- [7] I. Gaida, S. Mahapatra, T. Mohaupt and W. A. Sabra, Class. Quant. Grav. **16** (1999) 419 [arXiv:hep-th/9807014].
- [8] B. R. Greene, D. R. Morrison and A. Strominger, Nucl. Phys. B **451** (1995) 109 [arXiv:hep-th/9504145].
- [9] A. Strominger, Nucl. Phys. B **451** (1995) 96 [arXiv:hep-th/9504090].

- [10] B. R. Greene, D. R. Morrison and C. Vafa, Nucl. Phys. B **481** (1996) 513 [arXiv:hep-th/9608039].
- [11] A. Achucarro, M. de Roo and L. Huiszoon, Phys. Lett. B **424** (1998) 288 [arXiv:hep-th/9801082].
- [12] P. Candelas and X. C. de la Ossa, Nucl. Phys. B **342** (1990) 246.
- [13] T. Hubsch, “Calabi-Yau manifolds: A Bestiary for physicists,”
- [14] P. S. Green and T. Hubsch, Commun. Math. Phys. **119** (1988) 431.
- [15] P. S. Green and T. Hubsch, Phys. Rev. Lett. **61**, 1163 (1988).
- [16] P. Candelas, X. C. De La Ossa, P. S. Green and L. Parkes, Nucl. Phys. B **359** (1991) 21.
- [17] M. Bodner and A. C. Cadavid, Class. Quant. Grav. **7** (1990) 829.
- [18] M. Bodner, A. C. Cadavid and S. Ferrara, Class. Quant. Grav. **8** (1991) 789.
- [19] R. Bohm, H. Gunther, C. Herrmann and J. Louis, Nucl. Phys. B **569** (2000) 229 [arXiv:hep-th/9908007].
- [20] S. Gurrieri, arXiv:hep-th/0408044.
- [21] J. Polchinski, “String theory. Vol. 2: Superstring theory and beyond,”
- [22] D. Joyce, arXiv:math.dg/0111111.
- [23] K. Becker, M. Becker and A. Strominger, Nucl. Phys. B **456** (1995) 130 [arXiv:hep-th/9507158].
- [24] C. Vafa, Nucl. Phys. B **447** (1995) 252 [arXiv:hep-th/9505023].
- [25] A. Ceresole, R. D’Auria and S. Ferrara, Nucl. Phys. Proc. Suppl. **46** (1996) 67 [arXiv:hep-th/9509160].
- [26] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D’Auria, S. Ferrara, P. Fre and T. Magri, J. Geom. Phys. **23** (1997) 111 [arXiv:hep-th/9605032].
- [27] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D’Auria, S. Ferrara and P. Fre’, Nucl. Phys. B **476** (1996) 397 [arXiv:hep-th/9603004].
- [28] G. ’t Hooft, Phys. Rev. D **14** (1976) 3432 [Erratum-ibid. D **18** (1978) 2199].
- [29] B. de Wit and A. Van Proeyen, Nucl. Phys. B **245** (1984) 89.
- [30] L. Kofman, A. Linde, X. Liu, A. Maloney, L. McAllister and E. Silverstein, JHEP **0405** (2004) 030 [arXiv:hep-th/0403001].
- [31] T. W. B. Kibble, J. Phys. A **9** (1976) 1387.
- [32] H. B. Nielsen and P. Olesen, Nucl. Phys. B **61** (1973) 45.
- [33] T. Mohaupt and F. Saueressig, arXiv:hep-th/0410272.
- [34] T. Mohaupt and F. Saueressig, arXiv:hep-th/0410273.